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# MEASUREMENTS OF LOCAL TRANSFER COEFFICIENTS FOR DEVELOPING LAMINAR FLOW IN FLAT RECTANGULAR DUCTS

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**Abstract**—Local mass-transfer coefficients in the laminar entrance region of a flat rectangular duct were determined by application of the naphthalene sublimation technique. In accordance with the analogy between heat and mass transfer, the experimental conditions corresponded to a heat-transfer situation characterized by simultaneously developing velocity and temperature fields in an isothermal-walled parallel plate channel. Results were obtained for Reynolds numbers in the range between 350 and 1500. By use of a dimensionless streamwise coordinate involving the Reynolds number as a scaling factor, it was found that the data for the various Reynolds numbers could be brought together. The experimental results were compared with analytical predictions and good agreement prevailed. Overall mass balances performed using independent measurement procedures were satisfied to within a few per cent.

## NOMENCLATURE

- $D_e$, equivalent diameter, $2h$;
- $\mathscr{D}$, diffusion coefficient;
- $h$, duct height;
- $k$, mass-transfer coefficient;
- $\dot{m}$, local mass-transfer rate/area;
- $p_{nw}$, vapor pressure of naphthalene at wall;
- $Re$, Reynolds number, $\bar{u}D_e/\nu$;
- $Sh$, Sherwood number, $kD_e/\mathscr{D}$;
- $Sc$, Schmidt number;
- $T$, absolute temperature;
- $t_0$, duration of a data run;
- $\bar{u}$, mean velocity;
- $x$, streamwise coordinate;
- $\delta$, local sublimation depth;
- $\nu$, kinematic viscosity;
- $\rho_{ni}$, concentration of naphthalene vapor in incoming flow;
- $\rho_{nw}$, concentration of naphthalene vapor at duct wall;
- $\rho_s$, density of solid naphthalene.

## INTRODUCTION

THE PROBLEM of simultaneously developing laminar velocity and temperature fields in the entrance region of a parallel plate channel has evoked a number of analytical studies in the past two decades. Interest in the problem has been motivated by applications to compact heat exchangers. Owing to the complexity of the analysis, a variety of solution methods encompassing varying degrees of approximation were employed, including the Karman–Pohlhausen integral method [1, 2], the Langhaar–Han integral method [3, 4], series expansions [5, 6], and finite-differences [7, 8]. On the other hand, experimental investigation of the problem appears to be limited to the tests reported in [8]. The results presented therein tend to be rather sparse (e.g. eight local Nusselt number data points for the symmetrically heated channel) and restricted in range.

The present investigation was undertaken to provide more complete experimental information on the local transfer characteristics for simultaneously developing laminar velocity and temperature fields in a parallel plate channel. The experiments were performed using the naphthalene sublimation technique. In accordance with the analogy between heat and mass transfer, the present tests correspond to a heat-transfer problem with the same uniform surface temperature at each of the channel walls. A flat rectangular duct with a cross sectional aspect ratio of about 60 was used for the experimental runs, which encompassed a Reynolds number range from approximately 350–1500. Local transfer coefficients were determined as a function of a dimensionless axial coordinate which varied over a range of two orders of magnitude.

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## EXPERIMENTAL APPARATUS

The flat rectangular duct used in the experiments was made up of two parallel plates of naphthalene with side walls of plexiglass. The side walls served as spacers to fix the distance between the naphthalene plates. The resulting dimensions of the duct cross section were 1·5 × 88·3 mm (height by width), which corresponds to an aspect ratio of 58·9. The active length of the test section in the streamwise direction was about 65 mm.

At its upstream end, the duct mated with a slot that had been cut into a baffle plate, as shown schematically in Fig. 1 (the figure is not to scale). The front edges

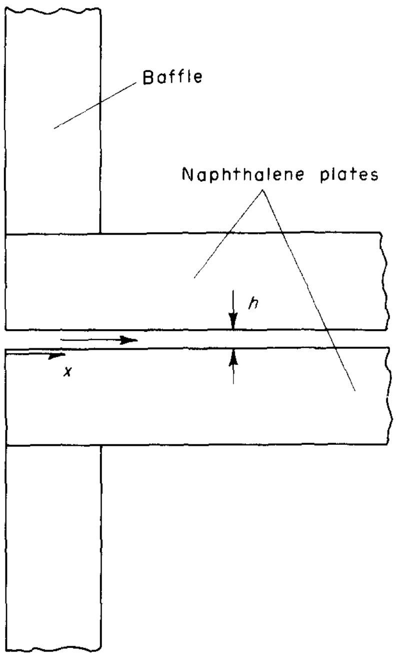


FIG. 1. Duct and baffle plate configuration.

of the duct walls were carefully aligned with the face of the baffle plate in order to provide a continuous surface. At its downstream end, the duct emptied into a plenum chamber which was connected to a flow meter via flexible hose. A blower situated downstream of the meter delivered the flow to an exhaust system, which discharged the naphthalene–air mixture at the roof of the building.

The placement of the blower downstream of the test section, rather than upstream, was purposeful. With the blower upstream, preheating of the air prior to its entry into the test section might have occurred, thereby causing a temperature rise. Since the vapor pressure of naphthalene is quite sensitive to temperature level (about 10 per cent/degC at room temperature), the preheating would have introduced an element of uncertainty in the results. Furthermore, the use of the building exhaust system ensured against the presence of naphthalene vapor in the room. Therefore, the air entering the test section was room temperature air, free of naphthalene vapor. The laboratory room itself was temperature-controlled, and the naphthalene plates, sealed between glass with a plastic outer wrap, were left in the room for a period of 24 h prior to the tests to ensure thermal equilibrium.

The naphthalene plates were cast in a specially designed mold. The metallic components of the mold, stainless steel plates and brass bars, had been hand polished and lapped to a mirror finish. When assembled, the mold enclosed a rectangular cavity whose top was left open for pouring the molten naphthalene. Once poured, the naphthalene was allowed to solidify under air cooling conditions. Removal of a cast plate was accomplished by hammer blows on strategic positions on the mold. Additional details of the mold construction and the casting procedure are described in [9].

The surfaces of the plates produced by the aforementioned casting procedure were extremely smooth and flat, so that further surface finishing operations were unnecessary. Furthermore, lubricants were not used to facilitate the removal of the plates from the mold. Extreme care was employed in cleaning all equipment involved with the casting procedure and in handling the cast plates subsequent to their removal from the mold. In view of these procedures and precautions, it can be assumed with confidence that the surfaces of the naphthalene plates were free of contamination.

A plate was never re-used subsequent to a data run. Rather, each new plate was cast from fresh naphthalene. To ensure against extraneous mass transfer during a data run, only that surface of the plate which was contacted by the duct flow was left exposed. All other surfaces were covered by a pressure sensitive tape.

The profile of the surface in the streamwise direction was measured both before a data run and after a data run by a precision dial gage. The smallest scale division on the dial gage was 0·00005 inches. It was mounted on a fixed strut that overhung a movable coordinate table. The coordinate table provided two directions of horizontal travel and was equipped with micrometer heads from which the horizontal position could be read to 0·002 mm. The naphthalene plates were held firmly against the coordinate table by flat springs.

The rate of flow through the test section was measured by a rotameter that had been calibrated by a volume displacement method (the calibration apparatus is described in [10]). Air temperature at the duct inlet was sensed by a calibrated copper–constantan thermocouple and recorded by a digital millivoltmeter. A laboratory timer was used to measure the duration of a data run.

## ANALYSIS OF DATA

Preliminary data runs were made to verify that the measurements were essentially independent of spanwise position. Surface profile measurements in the streamwise direction ($x$ direction, see Fig. 1) were made along the spanwise centerline and along lines displaced by 2·4 and 3·4 cm from the centerline. The measured profiles along these three lines were identical within the precision of the instrumentation. This outcome is entirely consistent with the predicted spanwise flatness of the velocity field for a 59:1 aspect-ratio rectangular duct. In fact, to encounter a 1 per cent spanwise variation in the velocity, it would be necessary to go to a spanwise position that is 4·2 cm from the centerline (half-span of duct = 4·4 cm).

For all of the final data runs, surface profile measurements were made along the line at 3·4 cm from the spanwise centerline. This choice was made to facilitate corrections for natural convection sublimation which occurred during the period when profiles were measured before and after a data run. The corrections were determined from surface profile measurements on that portion of the plate which was covered by the plastic side walls during the duration of a data run, but was uncovered during the measurement period. The proximity of the 3·4 cm spanwise location to the location of the free convection measurements was the main factor in its selection as the data collection site.

The surface profile measurements, corrected as described in the foregoing paragraph, yielded the streamwise distribution of the sublimation depth $\delta(x)$. Then, using the density $\rho_s$ of solid naphthalene ($\rho_s = 1{\cdot}145$ [11]) and the measured duration $t_0$ of the data run, the rate of mass transfer $\dot{m}$ per unit area was evaluated from

$$\dot{m}(x) = \rho_s \delta(x)/t_0. \quad (1)$$

A local mass-transfer coefficient $k$ can then be defined as

$$k = \frac{\dot{m}}{\rho_{nw} - \rho_{ni}} \quad (2)$$

where $\rho_{nw}$ and $\rho_{ni}$ are the concentrations (densities) of naphthalene vapor at the duct wall and in the flow entering the duct. For determining $\rho_{nw}$, it is necessary to employ the vapor pressure–temperature relation for naphthalene. There are several such relations in the literature and it is difficult to assess their relative merits. The Sogin correlation [12]

$$\log_{10} p_{nw} = 11{\cdot}884 - 6713/T \quad (3)$$

was ultimately adopted, where $p_{nw}$ is in lb/ft$^2$ and $T$ is in °R. With $p_{nw}$ from equation (3), $\rho_{nw}$ was evaluated from the perfect gas law. The concentration of naphthalene vapor in the entering flow was zero.

Then, a local Sherwood number can be evaluated from

$$Sh = kD_e/\mathscr{D} \quad (4)$$

in which $D_e$ is the equivalent diameter and $\mathscr{D}$ is the diffusion coefficient. For a parallel plate channel, $D_e = 2h$. The diffusion coefficient can be expressed in terms of the Schmidt number by $\mathscr{D} = \nu/Sc$. In view of the minute concentrations of naphthalene, $\nu$ was evaluated as that for pure air; in addition, $Sc = 2{\cdot}5$ [12].

The local Sherwood number results are to be presented as a function of a dimensionless streamwise coordinate

$$\frac{x/2h}{ReSc} \quad (5)$$

which is the reciprocal of the Graetz number, and $Re = \bar{u}D_e/\nu$.

In view of the fact that the channel height changed during the course of a data run as a result of sublimation, the quantity $h$ appearing in equations (4) and (5) was evaluated as the average of the initial and final heights. Generally, the average channel height differed by about 3 per cent from the initial height.

As a check on the instrumentation, experimental procedure, and data reduction technique, overall mass balances were performed. To this end, the sublimation depth profile $\delta(x)$ was integrated along the entire length of the plate. Upon multiplication of the integrated result by $\rho_s$ and by the exposed area of the plate, the amount of sublimed mass was determined. An alternative and fully independent measure of the sublimed mass was obtained by before and after weighing of the plate by means of a precision balance accurate to within 0·05 per cent in the range of the present measurements. Comparisons of the two sets of overall mass-transfer results will be made shortly.

## RESULTS AND DISCUSSION

The experimentally determined local Sherwood number results are plotted in Fig. 2 as a function of the dimensionless streamwise coordinate $(x/2h)/ReSc$. The figure contains the results for all of the five Reynolds numbers investigated, ranging from about 350–1500. The data for each Reynolds number are plotted separately in order to preserve clarity.

The ordinary axis actually serves as five different axes. The level of the ordinate for each set of data is fixed by indicating one Sherwood number for each set. The Sherwood number increment between hash marks

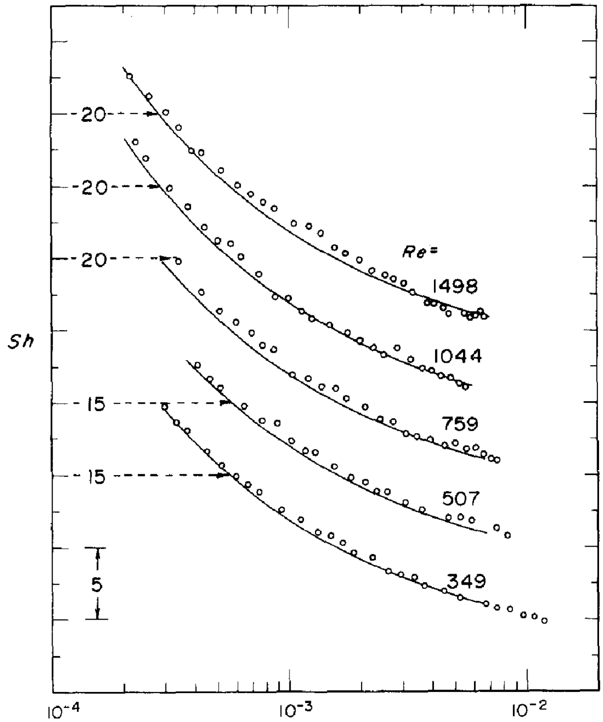


FIG. 2. Local Sherwood number results.

is given at the lower left of the figure. A relatively large scale has been used on the ordinate, so that both data scatter and deviations from analytical predictions tend to be exaggerated.

Associated with each set of data is a solid line. Actually, all of the solid lines are repetitions of the same line, each being drawn according to the ordinate scale that is specific for each Reynolds number. As will be discussed shortly, the lines represent analytical predictions, but for now they serve as convenient reference lines for making comparisons among the data for the various Reynolds numbers.

As seen in the figure, the transfer coefficients for each Reynolds number decrease monotonically with increasing distance from the duct inlet. Furthermore, the data for the various Reynolds numbers follow the respective reference lines with about the same degree of fidelity. Therefore, since the reference lines are identical, the data points for the various Reynolds numbers would fall together if they were plotted on the same ordinate scale. It may thus be concluded that the use of $(x/2h)/Re$ as a dimensionless streamwise coordinate successfully brings together the data for the different Reynolds numbers.* This finding is in accordance with published analyses, but must be accepted with the reservation that it is applicable only when the boundary layer assumptions are valid, that is, at sufficiently high Reynolds numbers.

*The Schmidt number is included in the abscissa variable of Fig. 2 in accordance with conventional practice, but a separate dependence on $Sc$ or $Pr$ still remains.

The quantitative outcome of the mass balances discussed at the end of the preceding section will now be presented. For the five Reynolds numbers, proceeding from lowest to highest, the respective deviations between the integral of the local mass transfer and the measured overall mass transfer were 1·3, 0·4, 0·5, 3·0 and 1·7 per cent. This excellent level of agreement lends support to the quality of the experimental apparatus and to the data acquisition procedures.

Analytical results for the isothermal-walled channel (by analogy, for uniform wall concentration) are provided in [1, 5–8], but only [8] presents results for the local transfer coefficients. The latter results are for $Pr = 0{\cdot}7$ and, therefore, cannot be employed for direct comparison with those of the present tests, which are for $Sc = 2{\cdot}5$. Fortunately, the average Nusselt number results of [1] are expressed by a relatively simple algebraic form, so that local transfer results can be obtained by differentiation with respect to the streamwise coordinate. The thus-obtained expression for the local Sherwood number is

$$Sh = 0{\cdot}332\,Pr^{1/3}\left(\frac{Re}{x/D_e}\right)^{1/2}\left[1+7{\cdot}3\left(\frac{x/D_e}{Re}\right)^{1/2}\right]^{1/2} \cdot \left\{1+\frac{3{\cdot}65\left(\frac{x/D_e}{Re}\right)^{1/2}}{1+7{\cdot}3\left(\frac{x/D_e}{Re}\right)^{1/2}}\right\}. \quad (6)$$

The analysis of [1] is based on the approximate Karman–Pohlhausen integral method and, therefore, so is equation (6). The range of applicability of equation (6) extends up to $(x/2h)/ReSc = 6\times10^{-3}$, which corresponds to the position where the boundary layers, growing from the two opposite walls of the channel, have met. To examine its validity, equation (6) has been evaluated for $Pr = 0{\cdot}7$ and compared with the finite difference results of [8]. It was found that equation (6) gives results that are 2·5–6 per cent high. Other comparisons cited in [8], all related to overall heat-transfer results, indicate a 6–7 per cent range of agreement among the various analytical predictions [1, 5, 7, 8].

The solid lines in Fig. 2 represent equation (6) with $Sc$ evaluated as 2·5. From an examination of the figure, it can be seen that for the most part, the data are within 5 per cent of the analytical prediction, although there are a few points which deviate by as much as 10 per cent. In general, the experimental results fall above the analytical prediction, and some consideration was given to possible reasons for this outcome.

Uncertainties in the Schmidt number and in the vapor pressure–temperature relationship might well be responsible for a few per cent of the just-cited deviation. Also considered was the possible effect of a sublimation-induced transverse velocity at the duct walls, but the corresponding variation of the Reynolds number along the channel was found to be negligible ($\Delta Re \ll 1$).

Still another potential factor might have been the contouring of the duct walls owing to the streamwise variation of the sublimation depth. During the course of a data run, a slight convergence of the walls was created by the sublimation. Very near the duct inlet, $(x/2h)/ReSc < 3 \times 10^{-4}$, it was estimated that a convergence with a half angle of about 3° developed during the duration of a run. Farther downstream, the convergence half angles were small fractions of a degree. Convergence does tend to increase the Sherwood number, but it would not be expected that convergence angles as small as the aforementioned would cause a 5–10 per cent effect.

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## MESURE DU COEFFICIENT LOCAL DE TRANSFERT POUR UN ECOULEMENT LAMINAIRE A L'ENTREE D'UNE CONDUITE RECTANGULAIRE ALLONGEE

**Résumé**—On détermine le coefficient local de transfert massique dans la région d'entrée laminaire d'une conduite rectangulaire allongée, en utilisant la technique de sublimation du naphtalène. En accord avec l'analogie entre transferts de masse et de chaleur, les conditions expérimentales correspondent à un transfert de chaleur caractérisé par des champs d'évolution de vitesse et de température dans un canal entre deux plans parallèles isothermes. Les résultats concernent les nombres de Reynolds compris entre 350 et 1500. A partir d'une coordonnée adimensionnelle, dans le sens du courant, introduisant le nombre de Reynolds comme un facteur d'échelle, on trouve que les résultats peuvent être rassemblés pour différents nombres de Reynolds. Ils sont comparés aux calculs et on obtient un bon accord. Des bilans globaux de masse établis par des méthodes de mesure indépendantes se recoupent à quelques pour cent près.

## MESSUNGEN VON ÖRTLICHEN AUSTAUSCHKOEFFIZIENTEN FÜR SICH AUSBILDENDE LAMINARE STRÖMUNG IN FLACHEN RECHTECKIGEN KANÄLEN

**Zusammenfassung**—Örtliche Stoffaustauschkoeffizienten in der laminaren Anlaufströmung von flachen rechteckigen Kanälen wurden mit Hilfe der Naphtalin-Sublimationstechnik bestimmt. In Übereinstimmung mit der Analogie zwischen Wärme- und Stoffaustausch entsprachen die experimentellen Bedingungen den Wärmeübergangsverhältnissen, die durch die gleichzeitige Ausbildung von Geschwindigkeits- und Temperaturfeldern in einem Kanal mit parallelen isothermen Wänden gekennzeichnet sind. Es wurden Ergebnisse für Reynolds-Zahlen zwischen 350 und 1500 gewonnen. Unter Verwendung einer dimensionslosen Koordinate, die die Reynolds-Zahl als einen Maßstabsfaktor enthält, ergab sich, daß die Ergebnisse für verschiedene Reynolds-Zahlen korreliert werden konnten. Die experimentellen Ergebnisse wurden mit analytischen Ansätzen verglichen. Es ergab sich gute Übereinstimmung. Stoffbilanzen, die unter Verwendung verschiedener Meßverfahren aufgestellt wurden, stimmten mit einer Abweichung von wenigen Prozent überein.

# ИЗМЕРЕНИЕ ЛОКАЛЬНЫХ КОЭФФИЦИЕНТОВ ПЕРЕНОСА ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ НА НАЧАЛЬНОМ УЧАСТКЕ ПЛОСКОГО ПРЯМОУГОЛЬНОГО КАНАЛА

**Аннотация** — С помощью методики сублимации нафталина определены локальные коэффициенты массопереноса на входном участке плоского прямоугольного канала. В соответствии с аналогией между тепло- и массопереносом экспериментальные условия соответствовали теплопереносу, характеризующемуся одновременно развивающимися полями скорости и температуры в плоско-параллельном канале с изотермическими стенками. Получены данные для чисел Рейнольдса в диапазоне от 350 до 1500. Найдено, что данные для различных чисел Рейнольдса могут быть обобщены с помощью безразмерной продольной координаты, включающей в себя в качестве масштабного фактора число Рейнольдса. Проведенное сравнение показало хорошее соответствие между экспериментальными данными и аналитическими расчетами. Общий баланс масс, выполненный с помощью независимых измерений, показал совпадение с точностью до нескольких процентов.